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A POSSIBLE ACOUSTICAL METHOD OF DETERMINING THE CONTENT OF AN OSCILLATING BUBBLE NEAR THE SEA FLOOR

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A possible acoustical method of determining the content of an oscillating bubble near the sea floor

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The Q value for a gas bubble oscillating in a liquid can be determined from the number of cycles required for the amplitude of the motion to decrease by a factor $e^{-1/2}$ as discussed below. The possibility of using the Q value in analysing the gas content of the bubble can be inferred from the results of Leighton and Walton,[1] in which they reported that the Q value of an air filled bubble (radius 1.7 mm oscillating in water) was found to be $Q = 37$, whereas a radius 0.87 mm of helium was Q=6.5. In view of the presence of bubbles emitted under various conditions at the sea floor, it is of interest to investigate the parameter Q in the case of, say, carbon dioxide and methane bubbles, and see if the distinction of Q value affords a means of identifying the bubbles gas content. The procedure would be based on the experiment described in reference 1. If not already oscillating, the bubble would be excited by an acoustic wave, and its response detected by hydrophones.

Q value can be defined in several ways. [2] A convenient definition for an oscillating system is:

$$Q = \text{the number of radians for the energy to decay to e}^{-1}$$

Thus, if n is the number of cycles for this decay, $Q = 2\pi n$. It is useful to relate quality factor Q to the kinematics of a damped oscillating bubble.

Following Leighton, the equations for the amplitude of the oscillation is:

$$m\ddot{x} + b\dot{x} + m\omega_0^2 x = 0$$

where x is the amplitude of the oscillation (considered small), m is the effective mass of the system; $\omega_0 = 2\pi\nu_0$ is the resonance frequency, and b is the resistive factor, describing

damping (loss of energy). Substituting $\beta = \frac{b}{2m}$ and assuming light damping ($\beta^2 << \omega_0^2$) the solution is

$$x = x_0 e^{i(\omega_0 t + \theta)}$$

where θ is a phaze factor and x_0 is the amplitude of the oscillation, which decays as $e^{-\beta t}$.

The logarithmic decrement is defined in terms of the log of the ratio of the amplitudes of 2 successive cycles:

$$\Delta_{log} = ln(x_{0,n}/x_{o,n+1}) = 2\pi\beta/\omega_0$$

The energy changes as x_0^2, so the ratio of the energies of 2 succesive cycles is

$$\left(\frac{x_0, n+1}{x_0, n}\right)^2 = e^{-2\Delta_{log}}$$

Thus, the number of cycles to reduce this by a factor of e is given by $2n\Delta_{log} = 1$ and the number of radians is

$$2\pi n = \frac{2\pi}{2\Delta_{log}} = \frac{\omega_0}{2\beta} = Q$$

To determine the Q value from the amplitude note that for the energy to decrease by e^{-1}, the amplitude ratio is

$$\left(\frac{x_n}{x_0}\right)^2 = e^{-1}$$

so that

$$\frac{x_n}{x_0} = e^{-1/2} = 0.61$$

Thus, an inspection of fig. 1 indicates a decrease in amplitude ~~by~~ to 0.6 close to 6 cycles, leading to

$$Q = 2\pi n \approx 38$$

The nature of the damping factor (here designated by β) requires further discussion.[2] For the applications anticipated in this work, the damping has been considered "light", i.e. $\beta << \omega_0$. Loss of energy can be due to radiation of sound,[1], or heat, for example, and these are discussed by Leighton.[2] For damping at the resonance frequency (as in the above) the damping constant δ is defined as

$$\delta = \frac{1}{Q}$$

It is important to note that the Q values reported in reference 1 were associated with bubble size. The relation between bubble size ρ_0 and resonance frequency ν_0 can be shown to be: [2]

$$\nu_0 = \frac{1}{2\pi R_0}\sqrt{\frac{3\kappa p_0}{\rho}}$$

where p_0 is the pressure of the liquid in which the bubble is oscillating; ρ is the density, p_0 is the pressure in atmospheres, and κ is the polytropic index in the gas law in the form pV^{κ}= constant.[1] For an isothermal process $\kappa = 1$, and for a reversible adiabatic process $\kappa = \gamma$, the ratio of specific heat at constant pressure is the specific heat at constant volume, and depends on the gas in the bubble. Taking $\gamma = 1.3$ the result is, for atmospheric pressure

$$R_0\nu_0 = 3.2\ m\ sec^{-1}$$

Anticipating the application of these relationships to experiments of bubbles of CO_2 and CH_4 on the sea bottom it must be noted that at 2500 m for example, $p_0 = 250$ atmospheres, so that

$$R_0\nu_0 \approx 50\ m\ sec^{-1}$$

Thus, for a bubble $R_0 = 1mm$, $\nu = 50kHz$, indicating the nature of the acoustic equipment necessary to excite and detect bubble signals.

It is possible that bubble sizing will be an important factor in Q value analyses and of interest in its own right. Walton [3,4] has called attention to techniques available for laboratory experiments to determine this parameter.

A combination frequency technique to measure the near surface bubble population in the open sea has been described by Phelps and Leighton.[5]

References

(1) Leighton, T.G.and A.J. Walton, An experimental study of the sound emitted from gas bubbles in a liquid, Eur. J. Phys. **8**, 98-104 (1987)

(2) Leighton, T.G., The Acoustic Bubble, Acadamic Press, 1994

(3) Leighton, T.G., R.J Lingard, A.J. Walton, and J.E. Field, Acoustic bubble sizing by combination of subharmonic emissions with imaging frequency Ultrasonics **29**, 319-323 (1991)

(4) Hardwick, A.J.and A.J.Walton, The acoustic bubble capacitor: a new method for sizing gas bubbles in liquids, Meas. Sci. Technol **6**, 202-205 (1995)

(5) Phelps, A.D. and T.G. Leighton, Oceanic bubble population measurements using a buoy-deployed combination frequency technique, IEEE Journal of Oceanic Engineering **23**, 400-410 (1998)

Figure caption

The hydrophone output showing the sound emitted from an air-filled bubble of radius 1.7 mm in water at 106 kPa. The signal from the hydrophone was fed through a broad-band amplifier (gain typically 30 dB) and to a transient recorder and pen recorder. (from Ref.1)

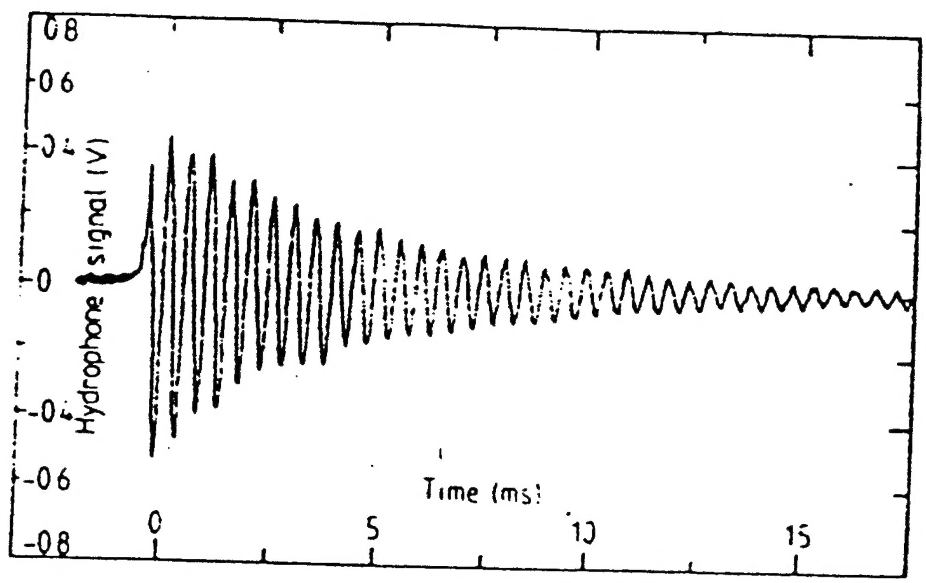

Figure 1

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14. ABSTRACT

A procedure is described in which acoustical methods can be used to study oscillating bubbles at the ocean floor. By observing the decay of the oscillation the Q value of the system can be measured. Laboratory experiments indicate that the Q value is a characteristic of the gas content of the bubble.

15. SUBJECT TERMS

Bubbles in the sea, bubble oscillation, bubble acoustics

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